



General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1	$\begin{array}{l} x \quad y \\ 1 \quad 0.5 \\ 3 \quad 0.366(0) \\ 5 \quad 0.309(0) \\ 7 \quad 0.274(3) \\ 9 \quad 0.25 \end{array}$ $\int = \frac{1}{3} \times 2 \times \left[\frac{(0.5+0.25)}{4(0.3660+0.2743)+2(0.3090)} \right]$ $= 2.62$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>4</p> <p>4</p>	<p>x values and no extra values</p> <p>4+ correct y values or $\frac{1}{1+\sqrt{3}}$ etc</p> <p>Correct application of Simpson's rule for their x values (x odd)</p> <p>CSO must be 3sf</p>
Total			4	
2	$V = (\pi) \int y^2 dx$ $= (\pi) \int (x-2)^5 dx$ $= (\pi) \left[\frac{(x-2)^6}{6} \right]_3^4$ $= (\pi) \left(\frac{2^6}{6} - \frac{1}{6} \right)$ $= 10.5\pi$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>4</p> <p>4</p>	<p>limits not required</p> <p>correct substitution into $(\pi)k(x-2)^6$</p> <p>allow equivalent fraction $\left(\frac{63}{6}\pi\right)$ etc (AWRT 10.5 or 10.5π m1, A0)</p>
Total			4	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$f(x) = x^3 + 5x - 4$ $f(0.5) = -1.375$ $f(1) = 2$ Change of sign $\therefore 0.5 < \alpha < 1$	M1 A1	2	Condone $f(0.5)$ rounding to -1.4 Both statements needed
(b)	$x^3 + 5x - 4 = 0$ $5x = 4 - x^3$ $x = \frac{1}{5}(4 - x^3)$	B1	1	Must be seen AG
(c)	$x_1 = 0.5$ $(x_2 = 0.775) (= \frac{31}{40})$ $x_3 = 0.707$	M1 A1	2	For x_2 or $x_3 = (2 \text{ sf})$
(d)		M1 A1	2	From 0.5 vertical to curve then horizontal to line CAO
Total			7	

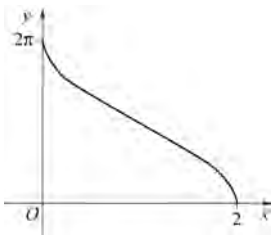
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\sec x = \frac{3}{2}$ $\cos x = \frac{2}{3}$ $x = 48, 312$ (Condone answers rounding to)	B1 B1	2	1 correct 2 correct and no extras in interval
(b)	$2 \tan^2 x = 10 - 5 \sec x$ $2(\sec^2 x - 1) = 10 - 5 \sec x$ $2 \sec^2 x + 5 \sec x - 12 (= 0)$ $(2 \sec x - 3)(\sec x + 4) (= 0)$ $\left. \begin{array}{l} \sec x = \frac{3}{2}, -4 \\ \cos x = \frac{2}{3}, -\frac{1}{4} \end{array} \right\} \text{either of these}$ $x = 48, 312, 104, 256$ Alternative: $\left. \begin{array}{l} \frac{2 \sin^2 x}{\cos^2 x} = 10 - \frac{5}{\cos x} \\ 2 \sin^2 x = 10 \cos^2 x - 5 \cos x \\ 2 - 2 \cos^2 x = 10 \cos^2 x - 5 \cos x \end{array} \right\}$ $12 \cos^2 x - 5 \cos x - 2 = 0$ then rest of scheme as above	M1 A1 m1 A1 B1 B1 (M1) (A1)	6	Use of trig identity correctly Attempt to solve or factorise 1 slip using formula AWRT 3 correct condone 105 or 255 All correct and no extras in interval
Total			8	
5(a)	$f(x) \leq 2, \quad f \leq 2, \quad y \leq 2$	B2	2	$\left. \begin{array}{l} \leq 2, f(x) < 2, x \leq 2 \\ y < 2, f < 2 \end{array} \right\} \text{B1}$
(b)	$f(x)$ is not one to one	E1	1	Allow many to one or numerical example
(c)(i)	$fg(x) = 2 - \left(\frac{1}{x-4}\right)^4$	B1	1	
(ii)	$2 - \left(\frac{1}{x-4}\right)^4 = -14$ $16 = \left(\frac{1}{x-4}\right)^4$ $\left. \begin{array}{l} (x-4)^4 = \frac{1}{16} \\ x-4 = \pm \frac{1}{2} \end{array} \right\}$ $x = 4\frac{1}{2}, 3\frac{1}{2}$	M1 M1 A1	3	Correct handling of fourth root Must have \pm Correct handling of reciprocal
Total			7	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$y = e^{2x}(x^2 - 4x - 2)$			
	$\frac{dy}{dx} = e^{2x}(2x - 4)$	M1		Product rule; allow 1 slip
	$+ (x^2 - 4x - 2)2e^{2x}$	A1		
	$\frac{dy}{dx} = e^{2x}(2x - 4 + 2x^2 - 8x - 4)$	M1		Factorising $e^{2x}(ax^2 + 6x + 0)$
	$e^{2x}(2x^2 - 6x - 8)$	A1		or $x^2 - 3x - 4 = 0$
	$e^{2x} \neq 0$ $(x - 4)(x + 1) = 0$ $x = 4, -1$	m1 A1	6	Solving 3 term quadratic Dependent on both M marks And no extras eg $x = 0$
(b)(i)	$\frac{d^2y}{dx^2} = e^{2x} \cdot 2 + (2x - 4)2e^{2x}$	M1		Product rule from their $\frac{dy}{dx}$ in form
	$+ (x^2 - 4x - 2)4e^{2x} + 2e^{2x}(2x - 4)$	A1		e^{2x} (quadratic) $e^{2x}(4x^2 - 8x - 22)$
	Or		2	
	$\frac{d^2y}{dx^2} = e^{2x}(4x - 6) + (2x^2 - 6x - 8)2e^{2x}$	M1 A1		
(ii)	$x = 4 : y'' = e^8(10) > 0 \therefore \text{MIN}$	M1		Their 2 x 's in their $\frac{d^2y}{dx^2}$
	$x = -1 : y'' = e^{-2}(-10) < 0 \therefore \text{MAX}$	A1	2	only of form e^{2x} (quadratic) CSO Both correct Allow values either side of y or y'
Total			10	
7(a)	$3e^x = 4$			
	$e^x = \frac{4}{3}$	M1		
(b)(i)	$x = \ln \frac{4}{3}$	A1	2	
	$3e^x + 20e^{-x} = 19$			
(ii)	$3y + \frac{20}{y} = 19$ or $3e^{2x} + 20 = 19e^x$			
	$3y^2 - 19y + 20 = 0$	B1	1	AG
	$(3y - 4)(y - 5) = 0$			
	$y = \frac{4}{3}, 5$	B1		
	$\therefore x = \ln \frac{4}{3}, \ln 5$	M1 A1	3	ln (their + ve y 's)
Total			6	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$P(-1, \pi)$ $Q(1, 0)$	B1 B1	2	Condone $(-1, 180^\circ)$
(b)	Translate $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Stretch SF 2 // y-axis	E1 B1 M1 A1	4	or equivalent in words Stretch + one other correct all correct
(c)		B1 B1	2	Correct shape in 1st quadrant 2π and 2 marked correctly
(d)(i)	$\frac{y}{2} = \cos^{-1}(x-1)$ $\cos\left(\frac{y}{2}\right) = x-1$ $x = \cos\left(\frac{y}{2}\right) + 1$	M1 A1	2	
(ii)	$-\frac{1}{2} \sin\left(\frac{y}{2}\right)$ At $y = 2$, $\left(\frac{dx}{dy}\right) = -\frac{1}{2} \sin 1$	M1 A1 A1	3	$k \sin (...)$ $\frac{dx}{dy}$ correct Condone AWRT -0.42
	Total		13	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$y = \frac{4x}{4x-3}$			
	$\frac{dy}{dx} = \frac{(4x-3) \cdot 4 - 4x(4)}{(4x-3)^2}$ $= \frac{-12}{(4x-3)^2}$	M1 A1	2	Must use quotient rule Condone one slip $k = -12$
(b)(i)	$y = x \ln(4x-3)$			
	$\frac{dy}{dx} = \frac{x \cdot 4}{4x-3} + \ln(4x-3)$	M1 m1 A1	3	$\frac{f(x)}{4x-3} + g(x)$ 'f(x)' may be constant $\frac{kx}{4x-3} + \ln(4x-3)$
(ii)	$x=1 \quad y=0$	B1		
	$\frac{dy}{dx} = 4$ $\therefore y = 4(x-1)$ any correct form	M1 A1	3	Sub $x=1$ into their $\frac{dy}{dx}$ CSO Must have full marks in (b)(i)
(c)(i)	$u = 4x-3$ $du = 4dx$	M1		
	$\int \frac{4x}{4x-3} dx = \int \frac{u+3}{u} \frac{du}{4}$ $= \left(\frac{1}{4}\right) \int \left(1 + \frac{3}{u}\right) (du)$ $= \frac{1}{4} (u + 3 \ln u)$ $= \frac{1}{4} [(4x-3) + 3 \ln(4x-3)] (+c)$	A1 m1 A1	4	Or $\int \frac{4x}{4x-3} dx = \int \left(1 + \frac{3}{4x-3}\right) dx$ $= \int \left(1 + \frac{3}{u}\right) du$ etc CSO Condone missing du
(ii)	$\int \ln(4x-3) dx$			
	$u = \ln(4x-3) \quad \frac{dv}{dx} = 1$ $\frac{du}{dx} = \frac{4}{4x-3} \quad v = x$ $\int = x \ln(4x-3) - \int \frac{4x}{4x-3} dx$ $= x \ln(4x-3) - \frac{1}{4} [(4x-3) + 3 \ln(4x-3)]$ (+c)	M1 A1 m1 A1	4	In correct direction $x \ln(4x-3) -$ their (c)(i)
Total			16	
TOTAL			75	